

Basic Rules for Algebra

I. Quadratic Equation:

The standard formula: $ax^2 + bx + c = 0$ (a, b, c are known values, $a \neq 0$; x is unknown variable)

Example: $x^2 + 5x + 6$ is a quadratic equation. $a=1, b=5, c=6$

$3x^2 + 5x$ is a quadratic equation; $a=3, b=5, c=0$

$5x + 7$ is NOT a quadratic equation because $a=0$ (there is no x^2)

To solve quadratic equations:

1. *Quadratic formula:* $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example: $x^2 - 5x + 6 = 0$. $a=1, b=5, c=6$

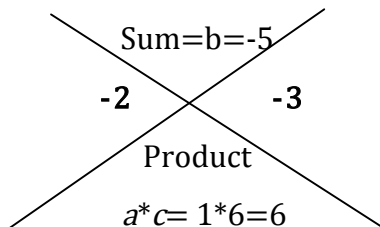
$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$ so $x = -2$ or $x = -3$

2. *Factoring by grouping or Diamond Method*

Find 2 numbers that have the Sum of b and the Product of a times c . After you get the two numbers, you will get the factors $(x + \text{constant 1})(x + \text{constant 2})$

Example: $x^2 - 5x + 6 = 0$

So we need to find 2 constants that have the sum of -5 and product of 6



$$x^2 - 5x + 6 = 0$$

$$(x + (-2))(x + (-3)) = 0$$

$$(x - 2)(x - 3) = 0$$

$$x - 2 = 0 \text{ or } x - 3 = 0$$

$$x = 2 \text{ or } x = 3$$



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II. Special Factors

Difference of Two Squares

$$x^2 - y^2 = (x - y)(x + y)$$

Example: $x^2 - 9 = (x - 3)(x + 3)$

Difference of Two Cubes

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Example: $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$

Sum of Two Cubes

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Example: $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$

III. Binomial Theorem

Definition: a quick way of expanding a binomial that is raised to any positive integer power

$$(x + y)^2 = x^2 + 2xy + y^2$$

Example: $(x + 2)^2 = x^2 + 2(x)(2) + 2^2 = x^2 + 4x + 4$

$$(x - y)^2 = x^2 - 2xy + y^2$$

Example: $(x - 2)^2 = x^2 - 2(x)(2) + 2^2 = x^2 - 4x + 4$

$$(x + y)^3 = x^3 + y^3 + 3xy^2 + 3x^2y + 27x + 9x^2$$

Example: $(x + 3)^3 = x^3 + 3^3 + 3(x)(3)^2 + 3(x)^2(3) = x^3 + 27 + 27x + 9x^2$

$$(x - y)^3 = x^3 - y^3 + 3xy^2 - 3x^2y + 27 + 27x - 9x^2$$

Example: $(x - 3)^3 = x^3 - 3^3 + 3(x)(3)^2 - 3(x)^2(3) = x^3 - 27 + 27x - 9x^2$

IV. Logarithmic Rules

Definition: Logarithms are the opposite of exponentials. With $a > 0$ and $a \neq 1$

$$\log_a x = y \text{ is equivalent to } x = a^y$$

Example: $\log_2 x = 3 \rightarrow x = 2^3 = 8$

Example: $10^3 = 1000 \rightarrow \log_{10} 1000 = 3$

Common Logarithm:

$$\log x = \log_{10} x$$

Natural Logarithm:

$$\ln x = \log_e x$$



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Properties of Logs

$$\log_a 1 = 0$$

Example: $\log_7 1 = 0$ (because $7^0 = 1$)

$$\log_a a = 1$$

Example: $\log_5 5 = 1$ (because $5^1 = 5$)

$$\log_a a^x = x$$

Example: $\log_5 5^8 = 8$ (because $5^8 = 5^8$)

$$a^{\log_a x} = x$$

Example: $5^{\log_5 12} = 12$

Laws of Logarithms

$$\log_a (AB) = \log_a A + \log_a B$$

Example: $\log_2(6x) = \log_2 6 + \log_2 x$

Example: $\log_4 2 + \log_4 32 = \log_4 (2 \cdot 32) = \log_4 64 = 3$

$$\log_a (A/B) = \log_a A - \log_a B$$

Example: $\log_5(x/3) = \log_5 x - \log_5 3$

Example: $\log_2 80 - \log_2 5 = \log_2(80/5) = \log_2 16 = 4$

$$\log_a A^c = c \log_a A$$

Example: $\log_3 x^2 = 2 \log_3 x$

Example: $3 \log_4 x = \log_4 x^3$

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References - The following works were referred to during the creation of this handout: Valle Verde Tutorial Support Service Handout.



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