Subject: Math and Statistics
Created by: Sunny Lin
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Permutation And Combination

Basic Definitions

Permutation --- **ORDERED** arrangement of
objects Combination --- **UNORDERED**
selections of objects

Permutation and Combination with DISTINCT objects

n = the number of all objects

r = the number of objects take out from n objects to do arrangement and selection

Type	*Repetition Allowed?*	*Formula*
r-Permutation	NO	$P(n,r) = \frac{n!}{(n-r)!}$
r-Permutation	YES	n^r
r-Combination	NO	$C(n,r) = \frac{n!}{r!(n-r)!}$
r-Combination	YES	$C(n+r-1,r) = \frac{(n+r-1)!}{r!\,(n-1)!}$

Permutations with INDISTINGUISHABLE objects

$$\frac{n!}{n_1!\, n_2! \cdots n_k!}$$

Different permutations of n objects, where there are n_1 INDISTINGUISHABLE objects of type 1, n_2 INDISTINGUISHABLE objects of type 2, …, n_k INDISTINGUISHABLE objects of type k.

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Example Questions

1. r-Permutation of n DISTINCT objects with NO REPETITION

If a repeated letter is not allowed in the string, how many 4-letter strings can be formed from the 26 uppercase English alphabets?

Solution: If any letter of a string changes, it makes a new string, so order matters. In an **ORDERED** counting, we use **PERMUTATION**. We are arranging 4 letter out of 26 alphabets, so **n=26**, **r=4**. Because **NO REPETITION** is allowed, if a letter is used in one slot of the string, it can't be used for the other slots. The choices of each letter within the 4-letter string is 26 25 24 23. The number of 4-letter strings of uppercase English alphabets will be $P(26,4) = \frac{26!}{(26-4)!} = \frac{26!}{22!}$

2. r-Permutation of n DISTINCT objects with REPETITION

How many 4-letter strings can be formed fr4m the uppercase English alphabet?

Solution: If any letter of a string changes, it makes a new string, so order matters. In **ORDERED** counting, we use **PERMUTATION**. We are arranging 4 letter out of 26 alphabets, so **n=26**, **r=4**. Because **REPETITION** is allowed, each letter can be any of the 26 alphabets. The choices of each letter within the 4-letter string is 26 26 26 26. The number of 4-letter strings of uppercase English alphabets is 26^4

3. Permutation with INDISTINGUISHABLE objects

How many different arrangements are there of the letters in the word "SUCCESS"?

Solution: If any letter of a string changes, it makes a new string, so order matters. In an **ORDERED** counting, we use **PERMUTATION**. We are RE-ARRANGING 7 letters within word "SUCCESS", so **n=r=7**. In word "SUCCESS", "$S_3UC_1C_2ES_1S_2$" and "$S_2UC_2C_1ES_3S_1$" are the same word. 3e two C's and three S's are **INDISTINGUISHABLE**, so we can't use the methods with distinct objects. There are 2 INDISTINGUISHABLE C's and 3 INDISTINGUISHABLE S's, so $n_1=2$ and $n_2=3$. The number of different arrangement is $\frac{n!}{n1!n2!}$ which is $\frac{7!}{2!3!}$

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4. r-Combination of n DISTINCT objects WITHOUT repetition

There is random picking of 5 numbers from 1 to 10, and each number can only be picked once. How many combinations can be formed?

Solution: For picking numbers, order doesn't matter. In an **UNORDERED** counting, we use **COMBINATION**. We are selecting 5 numbers out of 10 numbers, so **n=10**, **r=5**. Because **NO REPETITION** is allowed, if a number is already picked, it can't be picked again. The number of 5-number combinations picking from 1 to 10 will be $C(10,5) = \frac{10!}{(10-5)!} = \frac{10!}{5}$

5. r-Combination of n DISTINCT objects WITH repetition

Randomly picking 5 numbers from 1 to 10, and each number can be picked repeatedly. How many combinations can be formed?

Solution: For picking numbers, order doesn't matter. In an **UNORDERED** counting, we use **COMBINATION**. We are selecting 5 numbers out of 10 numbers, so **n=10**, **r=5**. Because **REPETITION** is allowed, even though a number is already picked, it can be picked again. The number of combinations will be $C(10,5) = \frac{10!}{(10-5)!} = \frac{10!}{5!}$

Steps to solve the problems

When you see a problem, please ask yourself the following questions:

1. Does ORDER matter in counting?
2. What is the n and r?
3. Any INDISTINGUISHABLE objects?
4. Repetition allowed?

References: The following works were referred to during the creation of this handout: *Discrete Mathematics and Its Applications.* 7th ed. Kenneth H. Rosen.