Subject: Math
Created by: Jiale (Olivia) Fan
Revised: 9/8/15

Basic Derivatives

Definition: The derivative of a function represents an infinitesimal change in the function with respect to one of its variables.

The "simple" derivative of a function f with respect to a variable x is $\frac{dy}{dx}$ denoted either $f'(x)$

1. Average Rate of Change

For $y = f(x)$, the average rate of change from $x = a$ to $x = a + h$ is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h} \quad h \neq 0$$

2. Instantaneous Rate of Change

For $y = f(x)$, the instantaneous rate of change at $x = a$ is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ if the limit exists

3. Slope of a Graph and Tangent Line

Given $y = f(x)$, the slope of the graph at the point $(a, f(a))$is given by

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

4. The Derivative

For $y = f(x)$, we define the derivative of f at x, denoted $f'(x)$,by

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If $f'(x)$ exists for each x in the open interval (a,b),then f is said to be differentiable over (a,b).

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Methods: Find the Derivative of a Function

There are two ways to do the Derivative:

Method 1: Using the definition of derivative

$$f'(x) \equiv \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

For example:

Find the derivative of $f(x) = x^2 + x$ using the definition of derivative

Solution: $f(x+h) = (x+h)^2 + (x+h)$

$$f(x) = x^2 + x$$

$$f'(x) \equiv \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, = \lim_{h \to 0} \frac{2x+2h+1-2x-1}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - (x^2+x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2+2xh+h^2+x+h-x^2-x}{h}$$

$$= \lim_{h \to 0} \frac{2xh+h^2+h}{h}$$

$$= \lim_{h \to 0} 2x + h + 1$$

Set $h = 0$

$$f'(x) = 2x + 1$$

(510) 885-3674
www.csueastbay.edu/scaa
scaa@csueastbay.edu

CALIFORNIA STATE UNIVERSITY EAST BAY

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Method 2: "Short-cut" way—power rule

Formula: $\frac{dy}{dx}x^n = nx^{n-1}$

Examples: Derivative for Common Functions:

Common Functions	Function	Derivative
Constant	c	0
	x	1
Square	x^2	2x
Square Root	$\sqrt{x}$	$(½)x^{-½}$
Exponential	e^x	e^x
	a^x	$a^x(\ln a)$
Logarithms	ln(x)	1/x
	$\log_a(x)$	1 / (x ln(a))

For Example:

Find the derivative of $f(x)= x^2$+x using the power rule.

Solution: $f'(x) = 2x^{2-1} + x^{1-1}$

$=2x + 1$ (Note that $x^0 = 1$)

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Reference: