Transformations of Functions

Vertical Shifting

Vertical Shifts of Graphs
Suppose \( c > 0 \).

To graph \( y = f(x) + c \), shift the graph of \( y = f(x) \) upward \( c \) units.
To graph \( y = f(x) - c \), shift the graph of \( y = f(x) \) downward \( c \) units.

Example

\[ f(x) = x^2 \]
\[ g(x) = x^2 + 3 \]
\[ h(x) = x^2 - 2 \]
Transformations of Functions

Horizontal Shifting

Horizontal Shifts of Graphs
Suppose $c > 0$.
To graph $y = f(x - c)$, shift the graph of $y = f(x)$ to the right $c$ units.
To graph $y = f(x + c)$, shift the graph of $y = f(x)$ to the left $c$ units.

Example

$g(x) = (x + 4)^2$  $f(x) = x^2$  $h(x) = (x - 2)^2$
Transformations of Functions

Reflecting Graphs

To graph \( y = -f(x) \), reflect the graph of \( y = f(x) \) in the x-axis.
To graph \( y = f(-x) \), reflect the graph of \( y = f(x) \) in the y-axis.

Examples
Transformations of Functions

Vertical Stretching and Shrinking

Vertical Stretching and Shrinking of Graphs

To graph $y = cf(x)$:

- If $c > 1$, stretch the graph of $y = f(x)$ vertically by a factor of $c$.
- If $0 < c < 1$, shrink the graph of $y = f(x)$ vertically by a factor of $c$.

Example

\[ f(x) = x^2 \]
\[ g(x) = 3x^2 \]
\[ h(x) = \frac{1}{3}x^2 \]
Transformations of Functions

Horizontal Stretching and Shrinking

**Horizontal Shrinking and Stretching of Graphs**

To graph $y = f(cx)$:

If $c > 1$, shrink the graph of $y = f(x)$ horizontally by a factor of $1/c$.

If $0 < c < 1$, stretch the graph of $y = f(x)$ horizontally by a factor of $1/c$.

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**Example**

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**References** --- The following work was referenced to during the creation of this handout: *Algebra and Trigonometry: Fourth Edition* (Stewart, Redlin, Watson).