Subject: Calculus
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Chain Rule

Background

The chain rule allows us to take the derivative of a composite function, meaning f(g(x)). For example, if y = sin(ln(x)) the outer function f(x) is sin(x) and the inner function g(x) is ln(x). Identifying the inner and outer functions is a crucial step in finding the derivative of a composite function. The following are some more examples:

- $y = \ln(x^2)$: outer function $f(x) = \ln(x)$ and inner function $g(x) = x^2$
- $y = e^{2x}$: outer function $f(x) = e^x$ and inner function $g(x) = 2x$
- $y = (x^4-5x)^3$: outer function $f(x) = x^3$ and inner function $g(x) = x^4-5x$

You try!

Find the outer and inner functions for the following:

1. $y = \tan^3(x)$ [Hint: $\tan^3(x) = (\tan(x))^3$]
2. $y = \log(3x^5 - e^x)$
3. $y = 2^{4x+5}$

Definition

Now that we have an understanding of composite functions, we can move on to the chain rule definition. The chain rule says that for a composite function y = f(g(x)), the derivative is

$y' = f'(g(x))*g'(x)$

For example, if y = sin(ln(x)), then y' = cos(ln(x))*1/x = cos(ln(x))/x. Here are some more examples:

- $y = \ln(x^2)$, $y' = (1/x^2)*\ d/dx(x^2) = (1/x^2)*\ (2x) = 2x/x^2 = 2/x$
- $y = e^{2x}$, $y' = e^{2x} * d/dx(2x) = e^{2x} * 2 = 2e^{2x}$
- $y = (x^4-5x)^3$, $y' = 3(x^4-5x)^2 * d/dx(x^4-5x) = 3(x^4-5x)^2(4x^3-5)$

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You try!

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1. $y = \tan^3(x)$ [Hint: $\tan^3(x) = (\tan(x))^3$]
2. $y = \log(3x^5 - e^x)$
3. $y = 2^{4x+5}$

Chain Rule With Complex Composite Functions

The chain rule can also be applied with more complex composite functions. For example, $y = \ln(\sin(e^{2x})$ is a composite function comprised of four functions: $\ln(x)$, $\sin(x)$, e^x, and $2x$. To find the derivative, apply the chain rule starting from the outer most function which in this case is $\ln(x)$.

$y = \ln(\sin(e^{2x}))$

$y' = [1/(\sin(e^{2x}))] * d/dx(\sin(e^{2x}))$

$= [1/(\sin(e^{2x}))] * \cos(e^{2x}) * d/dx(e^{2x})$

$= [1/(\sin(e^{2x}))] * \cos(e^{2x}) * (e^{2x}) * d/dx(2x)$

$= [1/(\sin(e^{2x}))] * \cos(e^{2x}) * (e^{2x}) * 2$

$= [2 * e^{2x} * \cos(e^{2x})]/\sin(e^{2x})$

You try!

Find the derivatives of the following:

1. $y = \cos^5(4x^5)$ [Hint: $\cos^5(x) = (\cos(x))^5$]
2. $y = (\log(e^{3x}))^5$
3. $y = \cot(6x^5 - 3x^2 + 5)^4$

Reference: Khan Academy