Subject: Statistics
Created by: Marija Stanojcic
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T-tests

- **One-sample t-test**: is used to estimate the mean of population; we have only ***one sample*** (we have a different handout for this test – *Statistical Tests for Population Mean)*

- **Two-sample t-test**: determines whether the means of ***two independent groups*** differ

- **Paired t-test**: determines whether the mean of the differences between ***two paired samples*** differs

Note: If you don't understand this handout, start with the Statistical Tests for Population Mean handout as there is a detailed explanation of how to conduct a test.

Two Sample T-Test

ASSUMPTIONS:

- *Populations distributions are normal*
- *Two samples are independent*

(Explanation: The observations from the first sample must not have any bearing on the observations from the second sample. For example, test scores of two separate groups of students are independent, but before-and-after measurements on the same group of students are not independent, although both of these examples have two samples.)

- Equal Variances:

H_0: **1.** $\mu_1 - \mu_2 \le D_0$ (D_0 is a specified value, often 0)
2. $\mu_1 - \mu_2 \ge D_0$
3. $\mu_1 - \mu_2 = D_0$

H_a: **1.** $\mu_1 - \mu_2 > D_0$
2. $\mu_1 - \mu_2 < D_0$
3. $\mu_1 - \mu_2 \ne D_0$

T.S.: $t = \dfrac{(\bar{y}_1 - \bar{y}_2) - D_0}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ $\quad s_p = \sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$

R.R.: For a level α, Type I error rate and with df $= n_1 + n_2 - 2$,
1. Reject H_0 if $t \ge t_\alpha$.
2. Reject H_0 if $t \le -t_\alpha$.
3. Reject H_0 if $|t| \ge t_{\alpha/2}$.

Check assumptions and draw conclusions.

Confidence interval: $(\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2} s_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$

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- Unequal Variances:

H_0: **1.** $\mu_1 - \mu_2 \le D_0$ H_a: **1.** $\mu_1 - \mu_2 > D_0$
2. $\mu_1 - \mu_2 \ge D_0$ **2.** $\mu_1 - \mu_2 < D_0$
3. $\mu_1 - \mu_2 = D_0$ **3.** $\mu_1 - \mu_2 \ne D_0$

T.S.: $$t' = \frac{(\bar{y}_1 - \bar{y}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

R.R.: For a level α, Type I error rate,
1. reject H_0 if $t' \ge t_\alpha$
2. reject H_0 if $t' \le -t_\alpha$
3. reject H_0 if $|t'| \ge t_{\alpha/2}$

with

$$\text{df} = \frac{(n_1 - 1)(n_2 - 1)}{(1 - c)^2(n_1 - 1) + c^2(n_2 - 1)}, \quad \text{and } c = \frac{s_1^2/n_1}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Note: If the computed value of df is not an integer, *round down* to the nearest integer.

Check assumptions and drawn conclusion.

Confidence interval: $$(\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Note: If the distribution isn't normal the nonparametric alternative tests can be done, for example – The Wilcoxon Rank Sum Test..

Paired T-Test

The paired observations are dependent and variances don't have to be equal.

(Explanation: Suppose managers at a fitness facility want to determine whether their weight-loss program is effective. Because the "before" and "after" samples measure the same subjects, a paired t-test is the most appropriate analysis.)

ASSUMPTIONS:

- *The differences need to be approximately normally distributed.*
- *The differences need to be independent*

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Calculate the difference between the two observations for each pair: $d_i = y_i - x_i$

Calculate the mean difference: $\bar{d} = \frac{d_1+d_2+ \ldots +d_n}{n}$

Calculate the standard deviation difference: $s_d = \sqrt{\frac{(d_1-\bar{d})^2+(d_2-\bar{d})^2+\ldots+(d_n-\bar{d})^2}{n}}$

Now, calculate t-test for sample differences: $t = \frac{\bar{d}-D_0}{\frac{s_d}{\sqrt{n}}}$

H_0: **1.** $\mu_d \leq D_0$ (D_0 is a specified value, often .0)
2. $\mu_d \geq D_0$
3. $\mu_d = D_0$

H_a: **1.** $\mu_d > D_0$
2. $\mu_d < D_0$
3. $\mu_d \neq D_0$

T.S.: $t = \frac{\bar{d} - D_0}{s_d/\sqrt{n}}$

R.R.: For a level α Type I error rate and with df $= n - 1$
1. Reject H_0 if $t \geq t_\alpha$.
2. Reject H_0 if $t \leq -t_\alpha$.
3. Reject H_0 if $|t| \geq t_{\alpha/2}$.

Check assumptions and draw conclusions.

Note: If the distribution isn't normal the nonparametric alternative tests can be done, for example – The Wilcoxon Signed – Rank Test..

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References: The following works were referred to during the creation of this handout: *Statistics: T-Tests SCAA handout by Farzaneh Roostaeigrailoo, An Introduction to Statistical Methods and Data Analysis, 6 th edition, Ott Longnecker;* *MiniTab's "Why Should I Use a Paired T-Test?"*, *Statistics Solutions "Paired T-Test Sample"*, *and* *Wikipedia's "One Sample T-Test" Article.*