Subject: Statistics
Created by: Marija Stanojcic
Revised: 07/10/2018

Statistical Tests for Population Mean (Z-test and T-test)

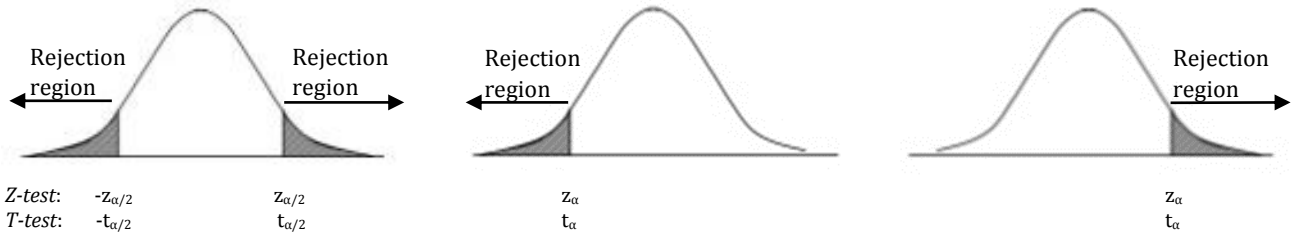
Step 1: Decide which test you need

Situation	Test	Notes
When the population is normal and σ is known.	Z-test	
When the population is normal but σ is unknown.	T-test	
When the population is non normal but the sample size is large enough (n ≥ 30) and σ is known.	Z-test	
When the population is non normal but the sample size is large enough (n ≥ 30) and σ is unknown.	Z-test	Replace σ by s; i.e. replace population standard deviation with sample standard deviation
When the population is non normal and the sample size is small (n < 30).		Beyond our scope at this point

Step 2: State the Hypothesis

Alternative hypothesis (H_a or H_1) is what the researcher wants to find out (what the researcher suspects).

Two-Tailed	Left-Tailed	Right-Tailed
$H_0: \mu = \mu_0$	$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$
$H_a: \mu \neq \mu_0$	$H_a: \mu < \mu_0$	$H_a: \mu > \mu_0$



Step 3: Calculate the Test Statistic

Depending on the test you chose in Step 1 calculate the appropriate test statistics.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \quad \text{or} \quad z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \text{ (for } \textit{large sample} \text{ and } \sigma \text{ is unknown)} \quad \text{or} \quad t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Note: $\bar{x}$ is the sample mean, and μ_0 is the mean from the hypothesis (with which we are comparing).

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Step 4: Decision Rule

a. p-value approach. Compute p-value, Reject H_0 when p-value < α.

Type of hypothesis	P-value (Z-test)	P-value (T-test)
Two sided: (Ha: $\mu \neq \mu_0$)	p-value = 2 · P(Z ≥ | computed z |)	p-value = 2· P(t ≥ | computed t |)
Left sided (Ha: $\mu < \mu_0$)	p-value = P(Z ≤ computed z)	p-value = P(t ≤ computed t)
Right sided (Ha: $\mu > \mu_0$)	p-value = P(Z ≥ computed z)	p-value = P(t ≥ computed t)

b. Critical value approach: Determine critical value(s) using α.

Type of hypothesis	Reject H_0 (Z-test)	Reject H_0 (T-test)
Two sided: (Ha: $\mu \neq \mu_0$)	$\|z\| > z_{\alpha/2}$ equivalent to $z > z_{\alpha/2}$ and $z < -z_{\alpha/2}$	$\|t\| > t_{\alpha/2}$ equivalent to $t > t_{\alpha/2}$ and $t < -t_{\alpha/2}$
Left sided (Ha: $\mu < \mu_0$)	$z < -z_{\alpha}$	$t < -t_{\alpha}$
Right sided (Ha: $\mu > \mu_0$)	$z > z_{\alpha}$	$t > t_{\alpha}$

Note: - If you are doing the T-test for calculating t_{α} you will also need number of degree of freedom (it is equal to sample size minus one, df = n – 1).

- For Z-test you can read value z_{α} /$z_{\alpha/2}$ from the Standard Normal Table, and for T-test you can find t_{α} /$t_{\alpha/2}$ from T table

Step 5: State the Conclusion

	Original Claim is H_0	Original Claim is H_1
Reject H_0	There is sufficient evidence (at the α level) to reject the claim that … .	There is sufficient evidence (at the α level) to support the claim that … .
Do Not Reject H_0	There is not sufficient evidence (at the α level) to reject the claim that … .	There is not sufficient evidence (at the α level) to support the claim that … .

Note: The level of significance is used to determine the critical value. The critical region includes the values of the shaded region. The shaded region is α. Using Z-table to find critical value.

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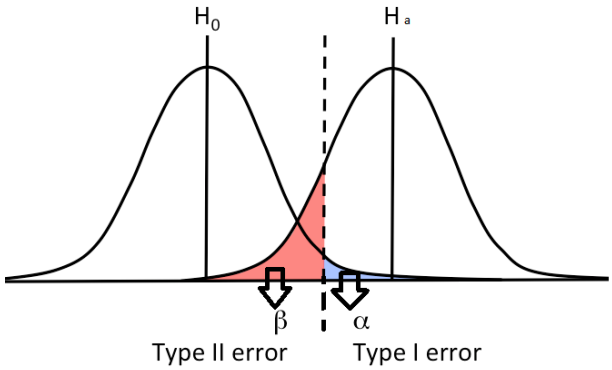
Two types of errors in decision making

Reality / Decision	**H_0 is True (Ha is False)**	**H_0 is False (Ha is True)**
Fail to Reject H_0 (Accept H_0)	Correct Decision	Type II error β
Reject H_0	Type I error α	Correct Decision Power = 1- β



Confidence interval: The (1− α)% confidence interval estimate for population mean is

Z-test: $\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ or $\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ (if large sample and unknown variance)

T-test: $\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

References: The following were referred to during the creation of this handout: *An Introduction to Statistical Methods and Data Analysis, 6th edition, Ott Longnecker; One Sample Z-Test and Confidence Interval For Estimating A Population Mean, SCAA Handout by Erica Yang; Statistics: Hypothesis Tests: When to use Which Test?, SCAA Handout by Gitanjali Shukla*