The Algebra of Lines

The Essential Forms of the Equation a Line

- Standard Form: $Ax + By = C$
- Slope-Intercept Form: $y = mx + B; \ m = \text{slope of line}, B = \text{y intercept of line.}$
- Point-Slope Form: $y - y_1 = m(x - x_1)$

Finding the Slope ($m$) of a Line Given 2 Points on the Line

- Given two points on a line $(x_1, y_1)$ and $(x_2, y_2)$

We use the equation: $\text{Slope} (m) = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$

**Example:**
Find the slope of a line that passes through the points $(1, 2)$ and $(3, 3)$

**Solution:**
$x_1 = 1, y_1 = 2, x_2 = 3 \text{ and } y_2 = 3$
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{3 - 1} = \frac{1}{2}$
$m = 1/2$

Finding the Equation of a Line Given 2 Points on the Line

- 1st find the slope ($m$) of the line: $\text{Slope} (m) = \frac{y_2 - y_1}{x_2 - x_1}$
- Next use the slope and any one of the given points to write the equation of the line using the Point-Slope formula: $y - y_1 = m(x - x_1)$ Or $y - y_2 = m(x - x_2)$.
- **Example 1:** Find the equation of a line that passes through the points $P_1(2,5)$ and $P_2(1,1)$.

**Solution:**
$x_1 = 2, y_1 = 5, x_2 = 1 \text{ and } y_2 = 1$
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 5}{1 - 2} = -\frac{4}{-1}$
$m = 4$

$y - y_1 = m(x - x_1)$
$y - 1 = 4(x - 1)$  Therefore  $y - 1 = 4x - 4$
$y = 4x - 3$

*Exercise: Try the same problem using $y - y_2 = m(x - x_2)$ you should get the same final answer.
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Finding the Equation of a Line Given the Slope and Point on the Line

- Follow step two of the procedure above [using the Point-Slope formula \( y - y_1 = m(x - x_1) \)] *Note sometimes a problem requires that we provide the equation of the required line in slope-intercept \((y = mx + B)\).
- For instance, in example 1 above, \( y - 1 = 4x - 4 \) is a perfectly good equation of the required line but \( y = 4x - 3 \) is the slope-intercept form of the equation of that line. Both equations describe the same line.

Parallel Lines

- If two lines \( L_1 \) and \( L_2 \) are parallel then their slopes, \( m_1 \) and \( m_2 \) are the same.
  \[ m_1 = m_2 \]
- **Example:** Find the slope of a line parallel to the line \( L: y = 2x + 1 \) and passes through the point \((1, 2)\).
  **Solution:**
  let slope of new line be \( m_n \) and slope of old line be \( m_o \)
  \[ m_n = m_o = 2 \]
  Using the point slope formula; \( y - y_1 = m(x - x_1) \)
  \[ y - 2 = 2(x - 1) \]
  \[ y = 2x - 2 \]
  \[ y = 2x \]

Perpendicular Lines

- If two lines \( L_1 \) and \( L_2 \) are perpendicular, then the product of their slopes is -1.
  \[ m_1 m_2 = -1, \quad m_1 = \frac{-1}{m_2}, \quad m_2 = \frac{-1}{m_1} \]
- **Example:** Find the equation of a line perpendicular to \( L: y = 2x + 1 \) that passes through the point \((1, 2)\).
  **Solution:**
  let slope of new line be \( m_n \) and slope of old line be \( m_o \)
  \[ m_n = ? , \quad m_o = 2 \]
  \[ m_n = \frac{-1}{m_o} = \frac{-1}{2} \]
  Using the point slope formula; \( y - y_1 = m(x - x_1) \)
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\[ y - 2 = \frac{-1}{2} (x - 1) \]
\[ y - 2 = \frac{-x}{2} + \frac{1}{2} \]
\[ y = \frac{-x}{2} - \frac{3}{2} \]
\[ y = -(x + 3) / 2 \]

**Intersecting Lines**

- Lines are said to intersect if they meet at some point on their profile. In the 2-Dimentional x-y plane, all non-parallel lines intersect.

  In order to find the point of intersection of any 2 lines, we equate them to each other.

- **Example:** Find the point of intersection of the lines \( L_1: y = 2x + 1 \) and \( L_2: y = x + 3 \)

  **Solution:**

  Equate the lines to each other

  \[ L_1 = L_2 \]

  \[ 2x + 1 = x + 3 \] for \( x \).

  \[ x = 2. \]

  After finding the value for \( x \), plug in this value into either \( L_1 \) or \( L_2 \) and solve for \( y \). If done correctly, you should obtain the same result for \( y \) from either equation.

  Using \( L_1: y = 2x + 1, \) \( x = 2 \rightarrow y = 2(2) + 1 = 5\)

  \[ x = 2, y = 5 \]

  Point of intersection is; \( (2, 5) \)

  Using \( L_2: y = x + 3, \) \( x = 2 \rightarrow y = 2 + 3 = 5\)

  \[ x = 2, y = 5 \]

  **Point of intersection** \( (2,5) \)