Subject: Algebra
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The Algebra of Lines

The Essential Forms of the Equation a Line

- Standard Form: $Ax + By = C$
- Slope-Intercept Form: $y = mx + B$; $m = slope\ of\ line, B = y\ intercept\ of\ line$.
- Point-Slope Form: $y - y_1 = m(x - x_1)$

Finding the Slope (m) of a Line Given 2 Points on the Line

- Given two points on a line $(x_1, y_1)\ and\ (x_2, y_2)$

  We use the equation: $Slope(m) = \frac{Rise}{Run} = \frac{y_2 - y_1}{x_2 - x_1}$
- **Example:**
  Find the slope of a line that passes through the points $(1, 2)\ and\ (3, 3)$
  Solution:
  $x_1 = 1$, $y_1 = 2$, $x_2 = 3$ and $y_2 = 3$
  $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{3 - 1} = \frac{1}{2}$$
  $m = 1/2$

Finding the Equation of a Line Given 2 Points on the Line

- 1st find the slope (m) of the line: $Slope(m) = \frac{y_2 - y_1}{x_2 - x_1}$
- Next use the slope and ***any one*** of the given points to write the equation of the line using the Point-Slope formula:
  $y - y_1 = m(x - x_1)$ Or $y - y_2 = m(x - x_2)$.
- **Example 1:** Find the equation of a line that passes through the points $P_1(2,5)\ and\ P_2(1,1)$.
  Solution:
  $x_1 = 2$, $y_1 = 5$, $x_2 = 1$ and $y_2 = 1$
  $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 5}{1 - 2} = \frac{-4}{-1}$$
  $m = 4$

  $y - y_1 = m(x - x_1)$
  $y - 1 = 4(x - 1)$ Therefore $y - 1 = 4x - 4$
  $y = 4x - 3$
  **Exercise: Try the same problem using $y - y_2 = m(x - x_2)$ you should get the same final answer.*

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Finding the Equation of a Line Given the Slope and Point on the Line

- Follow step two of the procedure above [using the Point-Slope formula $y - y_1 = m(x - x_1)$] **Note sometimes a problem requires that we provide the equation of the required line in slope-intercept ($y = mx + B$).*
- *For instance, in example 1 above, $y - 1 = 4x - 4$ is a perfectly good equation of the required line but $\mathbf{y = 4x - 3}$ is the slope-intercept form of the equation of that line. **Both equations describe the same line.***

Parallel Lines

- If two lines L_1 and L_2 are parallel then their slopes, m_1 and m_2 are the same.
  $m_1 = m_2$
- **<u>Example:</u>** Find the slope of a line parallel to the line $L: y = 2x + 1$ and passes through the point$(1, 2)$.
  <u>Solution:</u>
  $let\ slope\ of\ new\ line\ be\ m_n\ and\ slope\ of\ old\ line\ be\ m_o$
  $m_n = m_o = 2$
  Using the point slope formula; $y - y_1 = m(x - x_1)$
  $y - 2 = 2(x - 1)$
  $y - 2 = 2x - 2$
  $\underline{y = 2x}$

Perpendicular Lines

- If two lines L_1 and L_2 are perpendicular, then the product of their slopes is -1.
  $m_1 m_2 = -1,\ m_1 = \frac{-1}{m_2},\ m_2 = \frac{-1}{m_1}$
- **<u>Example:</u>** Find the equation of a line perpendicular to $L: y = 2x + 1$ that passes through the point $(1, 2)$
  <u>Solution</u>:
  $let\ slope\ of\ new\ line\ be\ m_n\ and\ slope\ of\ old\ line\ be\ m_o$
  $m_n = ?,\ m_o = 2$
  $m_n = \frac{-1}{m_o} = \frac{-1}{2}$
  Using the point slope formula; $y - y_1 = m(x - x_1)$

The Algebra of Lines

$y - 2 = \frac{-1}{2}(x - 1)$

$y - 2 = \frac{-x}{2} + \frac{1}{2}$

$y = \frac{-x}{2} - \frac{3}{2}$

$\underline{y = -(x + 3)/2}$

Intersecting Lines

- Lines are said to intersect if they meet at some point on their profile. In the 2-Dimentional x-y plane, all non-parallel lines intersect.

  In order to find the point of intersection of any 2 lines, we equate them to each other.

- **Example:** Find the point of intersection of the lines $L_1: y = 2x + 1$ and $L_2: y = x + 3$
  Solution:

  Equate the lines to each other

  $L_1 = L_2$

  $Solve$: $2x + 1 = x + 3$ for x.

  $x = 2$.

  After finding the value for x, plug in this value into either L_1 or L_2 and solve for y. If done correctly, you should obtain the same result for y from either equation.

  Using $L_1 : y = 2x + 1,\ x = 2 \rightarrow y = 2(2) + 1 = 5$

  $x = 2, y = 5$

  Point of intersection is; (2, 5)

  Using $L_2 : y = x + 3,\ x = 2 \rightarrow y = 2 + 3 = 5$

  $x = 2, y = 5$

  Point of intersection (2,5)