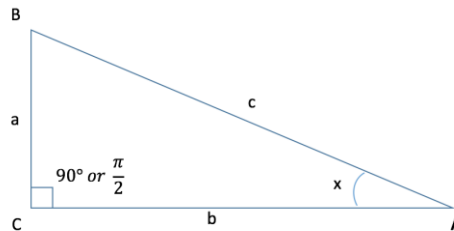


Shapes and Features of $\sin(x)$

We'll go through a very basic triangle function (\sin) to give you an idea of how to get the shape and use the shape for other questions. Other formulas will be very similar to these to.

1. Definitions:



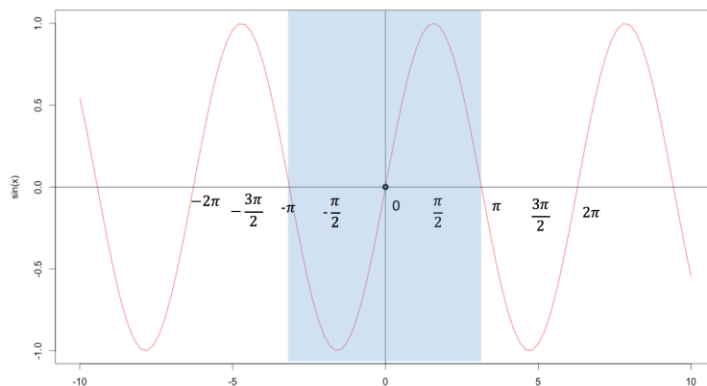
Pay attention, we always write the 'a' representing the line segment BC, which is also opposite to angle A (noted by ' $\angle BAC$ ')

$$y = \sin(x) = \frac{\textit{opposite}}{\textit{hypotenuse (longest one)}} = \frac{a}{c}$$

Exercise: To better understand this concept, I recommend you to fold this handout and try to draw this triangle yourself. After several tries and practice, you will have strong memory of this concept, which will be good for you to learn next topics.

2. Special points of triangle

$y = \sin(x)$: we can plot by changing the value of x , normally, you can just draw 5 points to get what the shape looks like: $x = -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$ (blue part below), other parts are repeating the blue part by a period of 2π :



Shapes and Features of $\sin(x)$

3. Here are some exercises you can do now. Try to work out the value of x below:

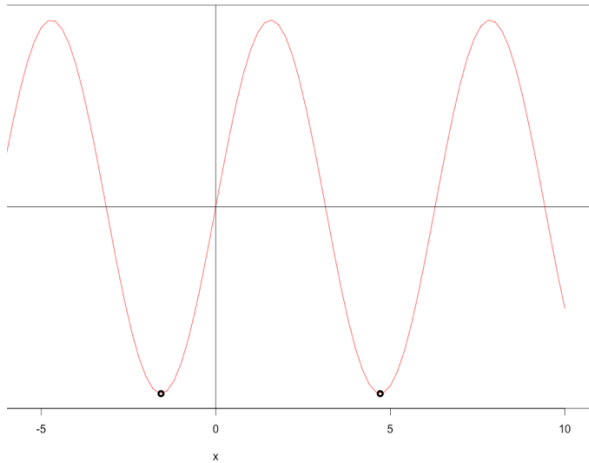
1) $\sin\left(x + \frac{\pi}{4}\right) = -\frac{1}{2}$, $0 \leq x \leq 2\pi$

2) $\sqrt{3}\sin\left(x + \frac{\pi}{4}\right) + \cos\left(x + \frac{\pi}{4}\right) = -1$, $0 \leq x \leq 2\pi$

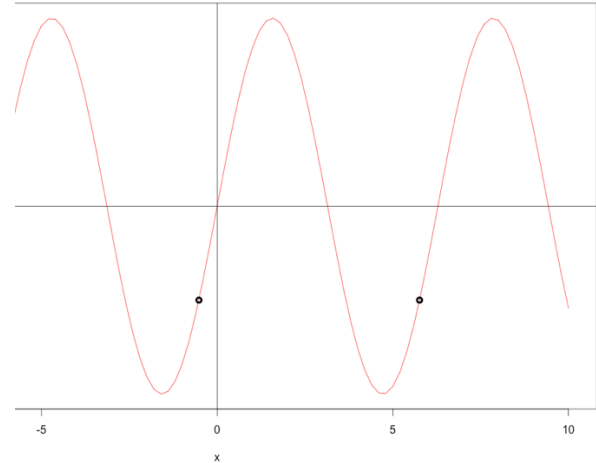
Solutions:

1) $\sin\left(x + \frac{\pi}{4}\right) = -1 \Rightarrow x + \frac{\pi}{4} = -\frac{\pi}{2} + 2k\pi$, k is any integer

Because $0 \leq x \leq 2\pi$, we can plot on axle to see which point of x can satisfy the condition:



plot for 1)



plot for 2)

So the only value satisfy $0 \leq x \leq 2\pi$ is $x = \frac{3}{2}\pi$

2) Note the constant $\sqrt{3}$, and 1 include we can use $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$ to simplify the equation.

$$\sqrt{3}\sin\left(x + \frac{\pi}{4}\right) + \cos\left(x + \frac{\pi}{4}\right) = -1 \Rightarrow \frac{\sqrt{3}}{2}\sin\left(x + \frac{\pi}{4}\right) + \frac{1}{2}\cos\left(x + \frac{\pi}{4}\right) = -\frac{1}{2}$$

$$\Leftrightarrow \cos\left(\frac{\pi}{6}\right)\sin\left(x + \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{6}\right)\cos\left(x + \frac{\pi}{4}\right) = -\frac{1}{2} \Rightarrow \sin\left(\frac{\pi}{6} + x + \frac{\pi}{4}\right) = -\frac{1}{2}$$

$$\Leftrightarrow x + \frac{5\pi}{12} = -\frac{\pi}{6} + 2k\pi, k \text{ is any integer. Because } 0 \leq x \leq 2\pi \text{ so } x = \frac{11}{6}\pi$$

