Subject: Algebra
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The Domain of a Function

Definition: *Domain of a function* is the set of "input" or argument values for which the function is defined.

How to find a domain of a given function?

Step 1: find points for which $f(x)$ is meaningless, which are always like:

- The numbers making denominator equals to 0
- The numbers making inside $\sqrt{x}$ less than 0
- The numbers out of definition for a certain function, e.g.: for y=In(x), x can not be less or equal to to 0

Step 2: the complementary set will be the domain

Example: Find out the domain of each following function:

1) $f(x) = \frac{1}{x(x-2)}$
2) $f(x) = \sin(x)$
3) $f(x) = \sqrt{x}$
4) $f(x) = \frac{e^x(x-3)\sqrt{x-7}}{x(x-2)\tan(x)}$

Solution:

1) the numbers make $f(x)$ meaningless are numbers making the denominator equals to 0, which is x=0 or x=2, so the rest numbers, all real numbers in $(-\infty, 0) \cup (0,2) \cup (2, +\infty)$ will be the answer. In another words, all real numbers after we skip x=0 and x=2 will make up the domain for $f(x) = \frac{1}{x(x-2)}$
2) no real number can make sin(x) meaningless, so the domain will be all real numbers
3) for a real function $f(x)$, x is not allowed to be less than 0, so oppositely, $x \geq 0$ will be allowed for $f(x) = \sqrt{x}$, which is the domain

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4) This one is much more complex than other questions, but don't worry, any complicated problem could be divided into small easy questions.

- First, look at the denominator, all 3 terms: x, x-2 and tan(x) can not be 0, and they themselves should be meaningful, so $x = 0, x = 2, x = \frac{\pi}{2} + k\pi (k = all\ integers)$ could not be involved in domain.
- Then let's turn to numerator, which has 3 terms: $e^x, x - 3, \sqrt{x-7}$:
  a. For e^x, x can be any real number
  b. For $x - 3$, x can be any number
  c. For $\sqrt{x-7}, x - 7\ should\ be\ non - negative\ number, which\ means, x - 7 \geq 0, or\ x \geq 7$
- Combine all results above, we get the domain for f(x), (Z means set of all integers):

$$\{x \neq 0, 2\ or \frac{\pi}{2} + k\pi, k\epsilon\ Z\ and\ x \geq 7\}$$

Summary:

Cases you should examine, which might make function meaningless:

1. Denominator can not be 0
2. Of any base, log(x) can only have x>0
3. Square root, like: $\sqrt{x-7}$ or root with even numbers, like: $\sqrt[4]{x}$, inner part must be larger or at least equal to 0
4. For triangle function:
   - sin(x) or cos(x) : x can be any real number
   - tan(x): x can not be: $\frac{\pi}{2} + k\pi, k\epsilon\ Z$, and for cot(x): x can not be $k\pi, k\epsilon\ Z$

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The following works were referred to during the creation of this handout: Wikipedia's entry on the domain of a function.