Subject: Statistics
Created by: Erica Yang
Revised: 3/17/2017

One Sample Z-Test and Confident Interval For Estimating A Population Mean

Step 1: Verify Assumptions

a. The sample is obtained using simple random sampling.

b. Sample size must be n > 30, population distribution must be normal with known variance. If variance is unknown, it can be approximated by sample variance S, then use Z-test.

Step 2: State the Hypothesis

Two-Tailed	Left-Tailed	Right-Tailed
H_0: $\mu = \mu_0$	H_0: $\mu \geq \mu_0$	H_0: $\mu \leq \mu_0$
H_1: $\mu \neq \mu_0$	H_1: $\mu < \mu_0$	H_1: $\mu > \mu_0$

Step 3: Calculate the Test Statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad \text{or} \quad z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \text{ (for } \textit{large sample} \text{ and } \sigma \text{ is unknown)}$$

Step 4: Decision Rule

a. p-value approach. Compute p-value, Reject H_0 when p-value < α.

Type of hypothesis	P-value
Two sided: if H_a : $\mu \neq \mu_0$	p-value = $2 \cdot P(Z \geq \lvert z \rvert)$
Left sided: if H_a : $\mu < \mu_0$	p-value = $P(Z \leq z)$
Right sided: if H_a : $\mu > \mu_0$	p-value = $P(Z \geq z)$

b. Critical value approach: Determine critical value(s) using α.

Type of hypothesis	Reject H_0
Two sided: if Ha : $\mu \neq \mu_0$	$\lvert z \rvert > Z_{\alpha/2}$ equivalent to $z > Z_{\alpha/2}$ and $z < - Z_{\alpha/2}$
Left sided: if Ha : $\mu < \mu_0$	$z < - Z_\alpha$
Right sided: if Ha : $\mu > \mu_0$	$z > Z_\alpha$

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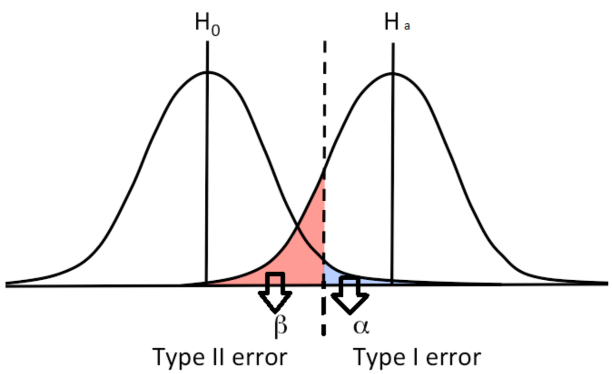
Step 5: State the Conclusion

	Original Claim is H_0	Original Claim is H_1
Reject H_0	There is sufficient evidence (at the α level) to reject the claim that … .	There is sufficient evidence (at the α level) to support the claim that … .
Do Not Reject H_0	There is not sufficient evidence (at the α level) to reject the claim that … .	There is not sufficient evidence (at the α level) to support the claim that … .

Note: The level of significance is used to determine the critical value. The critical region includes the values of the shaded region. The shaded region is α. Using Z-table to find critical value.

Two types of errors in decision making

Research Reality	**Null H_0 is True = Ha is False**	**Null H_0 is False = Ha is True**
Reject H_0	Type I error α	Correct Decision
Reject Ha	Power= 1-β	Type II error β



Confidence interval: The (1− α)% confidence interval estimate for population mean is

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \text{or} \quad \bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \quad \text{(if large sample and unknown variance)}$$