

# One Sample Z-Test and Confident Interval For Estimating A Population Mean

## Step 1: Verify Assumptions

- The sample is obtained using simple random sampling.
- Sample size must be  $n > 30$ , population distribution must be normal with known variance. If variance is unknown, it can be approximated by sample variance  $S$ , then use Z-test.

## Step 2: State the Hypothesis

Two-Tailed	Left-Tailed	Right-Tailed
$H_0: \mu = \mu_0$	$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$
$H_1: \mu \neq \mu_0$	$H_1: \mu < \mu_0$	$H_1: \mu > \mu_0$

## Step 3: Calculate the Test Statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad \text{or} \quad z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \quad (\text{for large sample and } \sigma \text{ is unknown})$$

## Step 4: Decision Rule

- p-value approach. Compute p-value, Reject  $H_0$  when p-value  $< \alpha$ .

Type of hypothesis	P-value
Two sided: if $H_a : \mu \neq \mu_0$	p-value = $2 \cdot P(Z \geq  z )$
Left sided: if $H_a : \mu < \mu_0$	p-value = $P(Z \leq z)$
Right sided: if $H_a : \mu > \mu_0$	p-value = $P(Z \geq z)$

- Critical value approach: Determine critical value(s) using  $\alpha$ .

Type of hypothesis	Reject $H_0$
Two sided: if $H_a : \mu \neq \mu_0$	$ z  > Z_{\alpha/2}$ equivalent to $z > Z_{\alpha/2}$ and $z < -Z_{\alpha/2}$
Left sided: if $H_a : \mu < \mu_0$	$z < -Z_{\alpha}$
Right sided: if $H_a : \mu > \mu_0$	$z > Z_{\alpha}$



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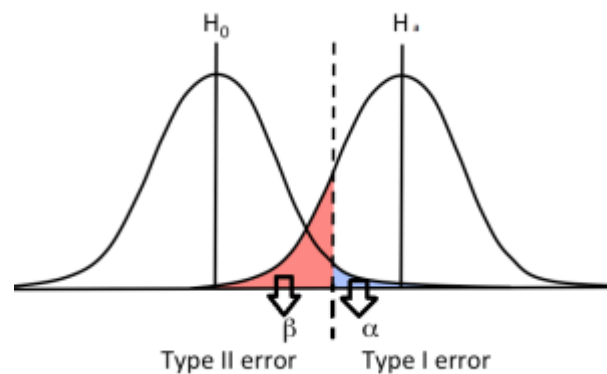
## Step 5: State the Conclusion

	Original Claim is $H_0$	Original Claim is $H_1$
<b>Reject <math>H_0</math></b>	There is sufficient evidence (at the $\alpha$ level) to reject the claim that ... .	There is sufficient evidence (at the $\alpha$ level) to support the claim that ... .
<b>Do Not Reject <math>H_0</math></b>	There is not sufficient evidence (at the $\alpha$ level) to reject the claim that ... .	There is not sufficient evidence (at the $\alpha$ level) to support the claim that ... .

**Note:** The level of significance is used to determine the critical value. The critical region includes the values of the shaded region. The shaded region is  $\alpha$ . Using Z-table to find critical value.

### Two types of errors in decision making

	Null $H_0$ is True = $H_a$ is False	Null $H_0$ is False = $H_a$ is True
Research Reality		
<b>Reject <math>H_0</math></b>	Type I error $\alpha$	Power= $1-\beta$
<b>Fail to Reject <math>H_0</math></b>	Correct Decision	Type II error $\beta$



**Confidence interval:** The  $(1 - \alpha)\%$  confidence interval estimate for population mean is

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \text{or} \quad \bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \quad (\text{if large sample and unknown variance})$$

