Subject: Calculus
Created by: Asees Kaur
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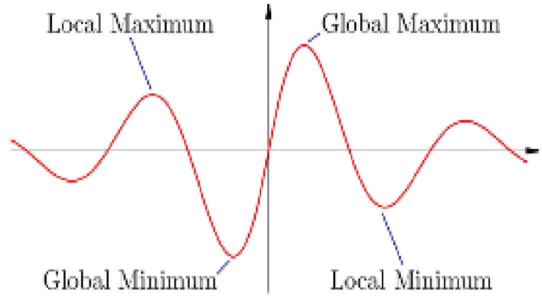
MAXIMA AND MINIMA



Absolute Maximum: Let f be defined on an interval I, and there exists a point c ∈ I. If f(c) ≥ f(x) for every x in I, then f has an **absolute maximum** value of f(c) on I at c.

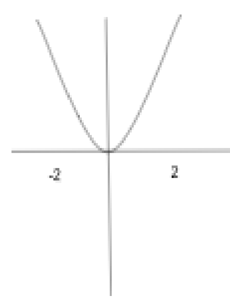
Absolute Minimum: Let f be defined on an interval I, and there exists a point c ∈ I. If f(c) ≤ f(x) for every x in I, then f has an **absolute minimum** value of f(c) on I at c.

Local Maximum: Let f be defined on an interval I, and there exists a point c ∈ I. If f(c) ≥ f(x) for every x in the open interval around c, then f(c)is the **local maximum** value of f.

Local Minimum: Let f be defined on an interval I, and there exists a point c ∈ I. If f(c) ≤ f(x) for every x in the open interval around c, then f(c)is the **local minimum** value of f.

Example:

Example: Locate the absolute and local maximum and minimum values, based on the picture.



Solution: Absolute maximum: at x = -2 and x = 2.

Absolute and local minimum: at x = 0

No local maximum.

Critical Points: Let f be defined on an interval I, and there exists a point c ∈ I(domain of f), then c is called a **critical point** if f'(c) = 0 or f'(c) does not exist.

Example: Find the absolute maximum and minimum values of the following functions.

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a.) $f(x) = x^4 - 2x^3$ on the interval [-2,2]

b.) $g(x) = x^{2/3}(2-x)$ on the interval [-1,2]

Solution:

a.) We know that f is a polynomial, thus it's derivative exists everywhere. Now let's find the critical points of f: $f'(x) = 0 \Rightarrow 4x^3 - 6x^2 = 0 \Rightarrow 2x^2(2x-3) = 0$

Solving this equation gives us two critical points: $x = 0$ and $x = 3/2$ and both of these points are in the interval [-2,2]. Now let's find the value of the function at the critical points and the end points to determine the absolute extrema.

$f(-2) = 32, \quad f(0) = 0, \quad f(3/2) = -27/16 \quad and \quad f(2) = 0$

Thus the given function has the largest value at $x = -2$ and the smallest value at $x = 3/2$.

Therefore, absolute maximum of f on [-2,2] is 32 and absolute minimum of f on [-2,2] is -27/16.

b.) Given: $g(x) = x^{2/3}(2-x) = 2x^{2/3} - x^{5/3}$. Now to find the critical points we will differentiate the function: $g'(x) = \frac{4}{3}x^{-1/3} - \frac{5}{3}x^{2/3} = (4-5x)/3x^{1/3}$. And we see that at $x = 0$, $g'(x)$ is undefined so we will consider $x = 0$ as a critical point too.

So, $g'(x) = 0 \Rightarrow 4 - 5x = 0 \Rightarrow x = \frac{4}{5}$. Thus we have two critical points: $x = 0$ *and* $x = \frac{4}{5}$. Now we will check the values of the function at the critical points and the end points, because we find the absolute extrema on the whole interval.

$g(-1) = 3, \quad g(0) = 0, \quad g(4/5) = 1.03 \quad and \quad g(2) = 0$. Thus we see that the function attains the largest value at $x = -1$ and the smallest at $x = 0$ *and* $x = 2$.

Therefore, absolute maximum of g on[-1,2] is 3 and absolute minimum of g on [-1,2] is 0.

References: Calculus for Scientists and Engineers,Volume 1 by William Briggs, Lyle Cochran, and Bernard Gillet; with assistance of Eric Schulz.