# Basic Rules for Algebra & Logarithmic Functions

## Quadratic Formula:

If \( p(x) = ax^2 + bx + c \), \( a \neq 0 \) and \( 0 \leq b^2 - 4ac \), then the real zeroes of \( p \) are \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

If \( p(x) = x^2 + 3x - 1 = 0 \), with \( a=1 \), \( b=3 \), and \( c=-1 \), then \( p(x) = 0 \) if \( x = \frac{-3 \pm \sqrt{13}}{2} \).

## Special Factors:

- \( x^2 - a^2 = (x - a)(x + a) \)
- \( x^3 - a^3 = (x - a)(x^2 + ax + a^2) \)
- \( x^3 + a^3 = (x + a)(x^2 - ax + a^2) \)

### Examples:

- \( x^2 - 9 = (x - 3)(x + 3) \)
- \( x^3 - 8 = (x - 2)(x^2 + 2x + 4) \)
- \( x^3 + 27 = (x + 3)(x^2 - 3x + 9) \)

## Binomial Theorem

- \((x + a)^2 = x^2 + 2ax + a^2\)
- \((x - a)^2 = x^2 - 2ax + a^2\)
- \((x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3\)
- \((x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3\)

### Examples:

- \((x + 3)^2 = x^2 + 2(3)(x) + 3^2 = x^2 + 6x + 9\)
- \((x - 5)^2 = x^2 - 2(5)(x) + 5^2 = x^2 - 10x + 25\)
- \((x + 2)^3 = x^3 + 3(2)(x^2) + 3(2)(x) + 2^3 = x^3 + 6x^2 + 12x + 8\)
- \((x - 1)^3 = x^3 - 3(1)(x^2) + 3(1)(x) - 1^3 = x^3 - 3x^2 + 3x - 1\)

## Factoring by Grouping

\[ ax^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d) \]

### Example:

\[ 3x^3 - 2x^2 - 6x + 4 = x^2(3x - 2) - 2(3x - 2) = (x^2 - 2)(3x - 2) \]

## Arithmetic Operations:

- \( ab + ac = a(b + c) \)
- \( \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \)
- \( \frac{a + b}{c} = \frac{a}{c} + \frac{b}{c} \)
- \( \frac{a}{b} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b} \frac{c}{c} \)

### Examples:

- \( \frac{a}{b} \cdot \frac{a}{b} = \frac{a}{b} \frac{c}{c} = \frac{ac}{b} \)
- \( \frac{a}{b} = \frac{a}{b} \cdot \frac{1}{1} = \frac{a}{b} \frac{1}{1} = \frac{a}{b} \)
- \( \frac{a}{b} = \frac{a}{b} \frac{1}{1} = \frac{1}{1} \frac{b}{b} = \frac{bc}{b} \)
- \( \frac{a}{b} = \frac{a}{b} \frac{c}{c} = \frac{a}{b} \frac{1}{1} = \frac{a}{bc} \)
Exponents and Radicals

\[
a^0 = 1, \ a \neq 0 \\
\frac{a^x}{a^y} = a^{x-y} \\
(a^x)^y = a^{xy} \\
\sqrt[n]{a^m} = a^{\frac{m}{n}} \\
\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \\
a^{-x} = \frac{1}{a^x} \\
(a^{-x})^y = a^{-xy} \\
\sqrt{a} = a^{\frac{1}{2}} \\
\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \\
a^x \cdot a^y = a^{x+y} \\
(ab)^y = a^y b^x \\
\sqrt[n]{a} = a^{\frac{1}{n}} \\
\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}
\]

Algebraic Errors to Avoid:

\[
\frac{a}{x+b} \neq \frac{a}{x} + \frac{a}{b} \\
\sqrt{x^2 + a^2} \neq x + a \\
a - b(x - 1) \neq a - bx - b \\
\frac{x}{a} \neq \frac{bx}{a} \\
\sqrt{-x^2 + a^2} \neq -\sqrt{x^2 - a^2} \\
a + bx \neq 1 + bx \\
(x^2)^3 \neq x^5
\]

To see this error, let \( a, b, \) and, \( x \) all equal to 1. Then, \( \frac{1}{2} \neq 2 \).

To see this error, let \( x = 3 \) and \( a = 4 \). Then, \( 5 \neq 7 \).

Remember to distribute the negative signs. The equation should be \( a - b(x - 1) = a - bx + b \).

To divide fractions, invert and multiply. The equation should be \( \frac{x}{a} = \frac{bx}{a} \).

We can’t factor a negative sign outside of the square root.

This is one of many examples of incorrect cancellation. By applying the properties above, the equation should be \( \frac{a + bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a} \).

In this case we multiply the exponents, then the correct equation is \( (x^2)^3 = x^2 \cdot x^2 = x^{2+2+2} = x^6 \).

LOGARITHMIC RULES

Logarithmic function
\[
y = \log_a x, \text{ defined by } x = a^y
\]

Common logarithm
\[
\log x = \log_{10} x
\]

Natural logarithm
\[
\ln x = \log_e x
\]

Log Properties
\[
\log_a a^x = x ; \ a^{\log_a x} = x
\]
\[
\log_a (MN) = \log_a M + \log_a N
\]
\[
\log_a (M/N) = \log_a M - \log_a N
\]
\[
\log_a (M)^y = y \log_a M
\]

Change of base formulas
\[
\log_b x = \frac{\log x}{\log b}
\]
\[
\log_b x = \frac{\ln x}{\ln b}
\]
Solving Quadratic Equations

Standard form of Quadratic Equation: \( ax^2 + bx + c = 0 \)

1. Completing the Square Method: Example: \( 2x^2 - 10x + 9 = 0 \)

   Step 1: Make sure the coefficient of \( x^2 \) is 1, if it is not already 1 then divide both sides of the equation by \( a \). For our example, we will divide by 2 on both sides, and we get: 
   \[ x^2 - 5x + \frac{9}{2} = 0. \]

   Step 2: Take the coefficient of \( x \) in the new equation in scratch, divide it by 2 and then square that term. So, it will look something like: 
   \[ \left( \frac{b}{2a} \right)^2, \] 
   we get: \[ \frac{25}{4}. \]

   Step 3: Now add \( \left( \frac{b}{2a} \right)^2 \) to both sides of the equation, and move the constant term on left side to the right side, so we get: 
   \[ x^2 - 5x + \frac{25}{4} = \frac{25}{4} - \frac{9}{2}. \]

   Step 4: Now you will notice that the left side of the equation becomes a perfect square, in our case we get: 
   \[ \left( x - \frac{5}{2} \right)^2 = \frac{7}{4}. \]

   Step 5: Now we will just solve this equation for \( x \), and here we get: 
   \[ x = \frac{5 \pm \sqrt{7}}{2}. \]

2. Factoring by grouping or Diamond Method

Find 2 numbers that have the Sum of \( b \) and the Product of \( a \) times \( c \). After you get the two numbers, you will get the factors \( (x + \text{constant 1}) \)(\(x + \text{constant 2}\))

Example: \( x^2 - 5x + 6 = 0 \)

So we need to find 2 constants that have the sum of -5 and product of 6

\[ \begin{array}{c|c|c}
\text{Sum} & \text{-2} & \text{-3} \\
\hline
\text{Product} & \text{2} & \text{6} \\
\hline
\times \text{c} & 1 \times 6 = 6 \\
\end{array} \]

\[ x^2 - 5x + 6 = 0 \]

\[ (x + (-2))(x + (-3)) = 0 \]

\[ (x - 2)(x - 3) = 0 \]

\[ x - 2 = 0 \text{ or } x - 3 = 0 \]

\[ x = 2 \text{ or } x = 3 \]

References - The following works were referred to during the creation of this handout: Valle Verde Tutorial Support Service Handout and Basic Rules for Algebra Handout by Tomee Lu (SCAA Tutor).