Subject: Calculus III
Created by: Ariel Li
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Testing Sequences and Series for Convergence

*** Some Suggestions About When & How to Use This Handout**
This handout is designed for **Math 2304 Calculus III** class as an abstract of useful techniques in Chapter 11 (Infinite Sequences and Series) of your textbook. ***Because this handout is very concise, it would better help you with specific examples from your textbook and/or class notes during your tutoring sessions.**

A. Sequence

Sequence	**Condition(s)**	**Limit for Convergence**	**Comment**
$f(n)=a_n$ $(n=1,2,3,\ldots)$	$\lim_{x\to\infty} f(x)$ = L	$\lim_{n\to\infty} a_n$ = L	
$f(n)=a_n$ $(n=1,2,3,\ldots)$	$\lim_{n\to\infty} \|a_n\|$ = 0	$\lim_{n\to\infty} a_n$=0	
$f(n)=a_n$ $(n=1,2,3,\ldots)$	$\lim_{n\to\infty} a_n$ = L and f is continuous at L	$\lim_{n\to\infty} f(a_n) = f(L)$	
$f(n)=\{r^n\}$ $(n=1,2,3,\ldots)$	**(i)** $if\ -1 < r < 1$; **(ii)** $if\ r = 1$	$\lim_{n\to\infty} r^n = \begin{cases} (i)\ 0 \\ (ii)\ 1 \end{cases}$	Otherwise, *f(n)* diverges

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B. Series

Test	**Series**	**Condition(s) of Convergence**	**Condition(s) of Divergence**	**Comment**	**Use the test if:**
***n*th-Term Test for Divergence**	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a^n \neq 0$	This test cannot be used to show convergence.	it can be seen at a glance that $\lim_{n\to\infty} a^n \neq 0$
Geometric Series	$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots$	$\|r\|<1$	$\|r\| \geq 1$	Sum: $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$	the series has the form $\sum ar^{n-1}$ or $\sum ar^n$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		Sum: $S = b_1 - L$	
***p*-series**	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p>1$	$0<p\leq 1$		the series has the form $\sum \frac{1}{n^p}$

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Alternating Series	$\sum_{n=1}^{\infty}(-1)^{n-1}b_n$	(i) $0 < b_{n+1} \leq b_n$ (ii) $\lim_{n\to\infty} b_n = 0$		Remainder: $\lvert R_n \rvert = \lvert S - S_n \rvert \leq b_{n+1}$	the series has the form $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$ or $\sum_{n=1}^{\infty}(-1)^{n}b_n$
Integral (*f* is <u>continuous,</u> <u>positive,</u> <u>decreasing</u> on [1, ∞])	$\sum_{n=1}^{\infty} a_n$, $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)dx$ converges	$\int_1^{\infty} f(x)dx$ diverges	Remainder: If $R_n = s - s_n$, $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$	$a_n=f(n)$, where $\int_1^{\infty} f(x)\,dx$ is easily evaluated
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt{\lvert a_n \rvert} = L < 1$	$\lim_{n\to\infty} \sqrt{\lvert a_n \rvert} = L > 1$ or $= \infty$	Test is inconclusive if $\lim_{n\to\infty} \sqrt{\lvert a_n \rvert} = L = 1$	a_n has the form $(b_n)^n$
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \lvert \frac{a_{n+1}}{a_n} \rvert = L < 1$	$\lim_{n\to\infty} \lvert \frac{a_{n+1}}{a_n} \rvert = L > 1$ or $= \infty$	Test is inconclusive if $\lim_{n\to\infty} \lvert \frac{a_{n+1}}{a_n} \rvert = L = 1$	Series that involve factorials (!) or other products
Direct Comparison $(a_n, b_n > 0)$	$\sum_{n=1}^{\infty} a_n$	$\sum_{n=1}^{\infty} b_n$ converges and $0 < a_n \leq b_n$	$\sum_{n=1}^{\infty} b_n$ diverges	Remainder: if $a_n \leq b_n$ and a_n & b_n converges;	the series has a form similar to a *p*-series or a

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			and 0 < $b_n \le a_n$	$R_n \le T_n$ (R_n = remainder of a_n, T_n = remainder of b_n)	geometric series, then compared with respective series; for Σa_n has some negative terms, apply the Comparison Test to $\Sigma\lvert a\rvert$ and test for absolute convergence
Limit Comparison $(a_n, b_n > 0)$	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n}$ = L > 0 and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n}$ = L > 0 and $\sum_{n=1}^{\infty} b_n$ diverges		

References: The Following works were referred to during the creation of this handout*: Calculus Early Transcendentals* (5E), *Calculus of a Single Variable* (9E)