

Basic Probability: Key Definitions and Rules

1. Key Probability Definitions and Notation

- **Probability** is a number between 0 and 1 that is assigned to a possible outcome of a random circumstance.
- A **simple event** is a unique possible outcome of a random circumstance.
- A **compound event** is an event that includes two or more simple events.
- An **event** is any collection of one or more simple events in the sample space; events can be simple events or compound events.
- Events are often written using capital letters A, B, C, and so on, and their probabilities are written as $P(A)$, $P(B)$, $P(C)$, and so on.
- One event is the **complement** of another event if the two events do not contain any of the same simple events and together they cover the entire sample space. For an event A, the notation A^c represents the complement of A.

E.g. Event A: The roll of a die is 2;

Event B: The roll of a die is NOT 2 (1 or 3-6).

Event B is the complement of Event A, or $P(B) = P(A^c)$

- Two events are **mutually exclusive** if they do not contain any of the same simple events (outcomes). Equivalent terminology is that the two events are **disjointed**.

E.g. Event A: The roll of a die is odd (1, 3, 5);

Event B: The roll of a die is even (2, 4, 6).

Event A and Event B are mutually exclusive.

- Two events are **independent** of each other if the probability of one occurring does not change the probability that the other occurs.

E.g. Event A: Roll a die, and the outcome is 1;

Event B: Roll another die, and the outcome is 1.

Event A and Event B are independent.



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- Two events are **dependent** if knowing that one will occur (or has occurred) changes the probability that the other occurs.

E.g. Event A: The roll of a die is odd (1, 3, 5);

Event B: The roll of a die is 1.

Event A and Event B are dependent.

- The **conditional probability of the event A, given that the event B has occurred or will occur**, is the long-run relative frequency with which event A occurs when circumstances are that B has occurred or will occur. It is written as $P(A/B)$.

E.g. Event A: The roll of a die is odd (1, 3, 5);

Event B: The roll of a die is not 1.

$P(A/B)$, or the **probability of Event A given B** means that given/assuming the roll of a die is not 1, the probability that it is odd (1, 3, 5).

2. Probability Rules

Rule 1 (for “not the event”): To find the probability of A^c , the complement of A, use

$$P(A^c) = 1 - P(A)$$

E.g. Event A: The roll of a die is 2; $P(A) = 1/6$

Event A^c : The roll of a die is NOT 2 (1 or 3-6). $P(A^c) = 1 - 1/6 = 5/6$

Rule 2 (addition rule for “either/or/and”): To find the probability that either A or B or both happen:

- Rule 2a (general):** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- Rule 2b (for mutually exclusive events):** If A and B are *mutually exclusive events*:

$$P(A \text{ or } B) = P(A) + P(B)$$

E.g. Event A: The roll of a die is odd (1, 3, 5); $P(A) = 3/6 = 1/2$

Event B: The roll of a die is even (2, 4, 6). $P(B) = 3/6 = 1/2$

Because Event A and Event B are mutually exclusive, $P(A \text{ or } B) = 1/2 + 1/2 = 1$



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Rule 3 (multiplication rule for “and”): To find the probability that two events, A and B , both occur simultaneously or in a sequence:

- **Rule 3a (general):** $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$
- **Rule 3b (for independent events):** If A and B are *independent events*,

$$P(A \text{ and } B) = P(A)P(B)$$

E.g. Event A: Roll a die, and the outcome is 1; $P(A) = 1/6$

Event B: Roll another die simultaneously, and the outcome is also 1. $P(B) = 1/6$

Because Event A and Event B are independent, $P(A \text{ and } B)$ (meaning that roll two dice simultaneously, and both outcomes are 2) = $1/6 * 1/6 = 1/36$

- **Extension of Rule 3b to more than two independent events:** For several *independent events*:

$$P(A_1 \text{ and } A_2 \dots \text{ and } A_{n-1} \text{ and } A_n) = P(A_1)P(A_2)\dots P(A_{n-1})P(A_n)$$

E.g. Event A_1 : Roll n dice simultaneously, and the outcome of the first die is 2; $P(A_1) = 1/6$

Event A_2 : Roll n dice simultaneously, and the outcome of the second die is 2. $P(A_2) = 1/6$

....

Event A_n : Roll n dice simultaneously, and the outcome of the n th die is 2. $P(A_n) = 1/6$

Because Event $A_1, A_2 \dots$ and A_n are independent of each other, $P(A_1 \text{ and } A_2 \dots \text{ and } A_{n-1} \text{ and } A_n) = 1/6 * 1/6 \dots 1/6 * 1/6 = 1/6^n$



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Rule 4 (conditional probability): To find the probability that B occurs given that A has occurred or will occur:

$$P(B|A) = P(A \text{ and } B)/P(A)$$

The assignment of letters to events A and B is arbitrary, so it is also true that

$$P(A|B) = P(A \text{ and } B)/P(B)$$

E.g. Event A: The roll of a die is odd (1, 3, 5); $P(A) = 1/2$

Event B: The roll of a die is not 1. $P(B) = 5/6$

$P(A \text{ and } B)$, or the probability that the roll of a die is odd and not 1 = the probability that the roll of a die is 3 or 5 = $2/6 = 1/3$

$$P(A|B) = P(A \text{ and } B) / P(B) = (1/3) / (5/6) = 2/5$$

3. Table for Reviewing the Rules:

When Events Are:	P(A or B) Is:	P(A and B) Is:	P (A B) Is:
Mutually Exclusive	$P(A) + P(B)$	0	0
Independent	$P(A) + P(B) - P(A)P(B)$	$P(A)P(B)$	$P(A)$
Any	$P(A) + P(B) - P(A)P(B)$	$P(A)P(B A)$	$P(A \text{ and } B)/P(B)$

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The following works were referred to during the creation of this handout: *Mind on Statistics (4th Edition)*.



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