Subject: Calculus
Created by: Alex Cheng
Revised: 8/5/2015

Derivative of Trig Functions

1) DERIVATIVE OF THE SINE FUNCTION

We will show that $\frac{d}{dx}\operatorname{Sin}(x) = \operatorname{Cos}(x)$	(1) We have $f(x) = \operatorname{Sin}(x)$
(2) Apply the limit definition: $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	$\lim_{h\to 0}\frac{\operatorname{Sin}(x+h)-\operatorname{Sin}(x)}{h}$
(3) Use the fundamental formulas of angle addition in trigonometry: $\operatorname{Sin}(x+h) = \operatorname{Sin}(x)\operatorname{Cos}(h)+\operatorname{Sin}(h)\operatorname{Cos}(x)$	$\lim_{h\to 0}\frac{\operatorname{Sin}(x)\operatorname{Cos}(h)+\operatorname{Sin}(h)\operatorname{Cos}(x)-\operatorname{Sin}(x)}{h}$
(4) Group $[\operatorname{Sin}(x)\operatorname{Cos}(h)]$ and $[-\operatorname{Sin}(x)]$ together then factor out $[\operatorname{Sin}(x)]$	$\lim_{h\to 0}\frac{\operatorname{Sin}(x)[\operatorname{Cos}(h)-1]+\operatorname{Cos}(x)\operatorname{Sin}(h)}{h}$
(5) Since both terms share the same common denominator h, we can now split the numerator into two terms.	$\lim_{h\to 0}\left(\operatorname{Sin}(x)\left(\frac{\operatorname{Cos}(h)-1}{h}\right)+\operatorname{Cos}(x)\left(\frac{\operatorname{Sin}(h)}{h}\right)\right)$
(6) Distribute $\left(\lim_{h\to 0}\right)$	$\lim_{h\to 0}\operatorname{Sin}(x)\times\lim_{h\to 0}\left(\frac{\operatorname{Cos}(h)-1}{h}\right)+\lim_{h\to 0}\operatorname{Cos}(x)\times\lim_{h\to 0}\left(\frac{\operatorname{Sin}(h)}{h}\right)$
(7) Since $\lim_{h\to 0}\left(\frac{\operatorname{Cos}(h)-1}{h}\right) = 0$ and $\lim_{h\to 0}\left(\frac{\operatorname{Sin}(h)}{h}\right) = 1$ In addition, $\lim_{h\to 0}\operatorname{Sin}(x) = \operatorname{Sin}(x)$ and $\lim_{h\to 0}\operatorname{Cos}(x) = \operatorname{Cos}(x)$	$= \operatorname{Sin}(x)\times 0+\operatorname{Cos}(x)\times 1$ $= 0+\operatorname{Cos}(x)$ $= \operatorname{Cos}(x)$
(8) Therefore, $\frac{d}{dx}\operatorname{Sin}(x) = \operatorname{Cos}(x)$	

2) DERIVATIVE OF THE COSINE FUNCTION

We will show that $\frac{d}{dx}\operatorname{Cos}(x) = -\operatorname{Sin}(x)$	(1) We have $f(x) = \operatorname{Cos}(x)$
(2) Apply the limit definition: $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	$\lim_{h\to 0}\frac{\operatorname{Cos}(x+h)-\operatorname{Cos}(x)}{h}$
(3) Use the fundamental formulas of angle addition in trigonometry:	$\lim_{h\to 0}\frac{\operatorname{Cos}(x)\operatorname{Cos}(h)-\operatorname{Sin}(x)\operatorname{Sin}(h)-\operatorname{Cos}(x)}{h}$

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$\text{Cos}(x + h) \quad \text{Cos}(x)\text{Cos}(h) \quad \text{Sin}(x)\text{Sin}(h)$	
(4) Group $[\text{Cos}(x)\text{Cos}(h)]$ and $[\ \text{Cos}(x)]$ together then factor out $[\text{Cos}(x)]$	$\lim_{h \to 0} \frac{\text{Cos}(x)[\text{Cos}(h) \quad 1] \quad \text{Sin}(x)\text{Sin}(h)}{h}$
(5) Since both terms share the same common denominator h, we can now split the numerator into two terms.	$\lim_{h \to 0} \left(\text{Cos}(x) \left(\frac{\text{Cos}(h) \quad 1}{h} \right) \quad \text{Sin}(x) \left(\frac{\text{Sin}(h)}{h} \right) \right)$
(6) Distribute $\left(\lim_{h \to 0} \right)$	$\lim_{h \to 0} \text{Cos}(x) \times \lim_{h \to 0} \left(\frac{\text{Cos}(h) \quad 1}{h} \right) \quad \lim_{h \to 0} \text{Sin}(x) \times \lim_{h \to 0} \left(\frac{\text{Sin}(h)}{h} \right)$
(7) Since $\lim_{h \to 0} \left(\frac{\text{Cos}(h) - 1}{h} \right) \quad 0$ and $\lim_{h \to 0} \left(\frac{\text{Sin}(h)}{h} \right) \quad 1$ In addition, $\lim_{h \to 0} \text{Sin}(x) \quad \text{Sin}(x)$ and $\lim_{h \to 0} \text{Cos}(x) \quad \text{Cos}(x)$	$\text{Cos}(x) \times 0 \quad \text{Sin}(x) \times 1$ $0 \quad \text{Sin}(x)$ $\text{Sin}(x)$
Therefore, $\frac{d}{dx} \text{Cos}(x) \qquad \text{Sin}(x)$	

3) DERIVATIVE OF THE TANGENT FUNCTION

We will show that $\frac{d}{dx} \text{Tan}(x) \quad \text{Sec}^2(x)$	(1) We have $f(x) \quad \text{Tan}(x)$
(2) Use the trig identity for Tangent	$\text{Tan}(x) \quad \frac{\text{Sin}(x)}{\text{Cos}(x)}$
(3) Apply the Quotient rule	$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \times \frac{d}{dx} f(x) - f(x) \times \frac{d}{dx} g(x)}{(g(x))^2}$
(4) From (1) and (2), we know that the derivative of $\text{Sin}(x) \quad \text{Cos}(x)$ and $\text{Cos}(x) \quad \text{Sin}(x)$	$\frac{d}{dx} \left(\frac{\text{Sin}(x)}{\text{Cos}(x)} \right) \quad \frac{\text{Cos}(x) \times \text{Cos}(x) \quad \text{Sin}(x) \times \quad \text{Sin}(x)}{(\text{Cos}(x))^2}$
(5) Simplify	$\frac{\text{Cos}^2(x) + \text{Sin}^2(x)}{\text{Cos}^2(x)}$
(6) Use the trig identity under Pythagorean Identities $\text{Sin}^2(x) + \text{Cos}^2(x) \quad 1$	$= \frac{1}{\text{Cos}^2(x)} = \frac{1}{\text{Cos}(x)} \times \frac{1}{\text{Cos}(x)}$
(7)Use the trig identity $\text{Sec}(x) \quad \frac{1}{\text{Cos}(x)}$	$= \text{Sec}(x) \times \text{Sec}(x) \quad \text{Sec}^2(x)$
Therefore, $\frac{d}{dx} \text{Tan}(x) \quad \text{Sec}^2(x)$	

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4) DERIVATIVE OF THE COSECANT FUNCTION

We will show that $\frac{d}{dx}Csc(x) \quad Csc(x)Cot(x)$	(1) We have $f(x) \quad Csc(x)$
(2) Use the trig identity for Tangent	$Csc(x) \quad \frac{1}{Sin(x)} \quad (Sin(x))^{\ 1}$
(3) Apply the Chain rule: $\frac{d}{dx}f(g(x)) \times \frac{d}{dx}g(x)$	$\frac{d}{dx}((Sin(x))^{\ 1}) = -1((Sin(x))^{\ 2}(Cos(x))$
(4)Simplify	$-\frac{1}{((Sin(x))^2} \times \frac{Cos(x)}{1} = -\frac{Cos(x)}{Sin^2(x)}$
(5) Use the trig identity $Csc(x) \quad \frac{1}{Sin(x)}$ and $Cot(x) \quad \frac{Cos(x)}{Sin(x)}$	$= -\frac{Cos(x)}{Sin(x)} \times \frac{1}{Sin(x)} = -Cot(x)\,Csc(x)$
Therefore, $\frac{d}{dx}Csc(x) \quad Csc(x)Cot(x)$	

5) DERIVATIVE OF THE SECANT FUNCTION

We will show that $\frac{d}{dx}Sec(x) \quad Sec(x)Tan(x)$	(1) We have $f(x) \quad Sec(x)$
(2) Use the trig identity for Tangent	$Sec(x) \quad \frac{1}{Cos(x)}$
(3) Apply the Chain rule	$\frac{d}{dx}((Cos(x))^{\ \cdot}) = -1((Cos(x))^{\ 2}(\ Sin(x))$
(4)Simplify	$= -\frac{1}{((Cos(x))^2} \times -\frac{Sin(x)}{1} = \frac{Sin(x)}{Cos^2(x)}$
(5) Use the trig identity $Tan(x) \quad \frac{Sin(x)}{Cos(x)}$ and $Sec(x) \quad \frac{1}{Cos(x)}$	$= \frac{Sin(x)}{Cos(x)} \times \frac{1}{Cos(x)} = Tan(x)Sec(x)$
Therefore, $\frac{d}{dx}Sec(x) \quad Sec(x)Tan(x)$	

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6) DERIVATIVE OF THE COTANGENT FUNCTION

We will show that $\frac{d}{dx}\text{Cot}(x) \quad \text{Csc}^2(x)$	(1) We have $f(x) \quad \text{Cot}(x)$
(2) Use the trig identity for Tangent	$\text{Cot}(x) \quad \frac{\text{Cos}(x)}{\text{Sin}(x)}$
(3) Apply the Quotient rule	$\frac{d}{dx}\left(\frac{\text{Cos}(x)}{\text{Sin}(x)}\right) \quad \frac{\text{Sin}(x)\times \quad \text{Sin}(x) \quad \text{Cos}(x)\times\text{Cos}(x)}{(\text{Sin}(x))^2}$
(4)Simplify	$\frac{\text{Sin}^2(x) \quad \text{Cos}^2(x)}{\text{Sin}^2(x)} \quad \frac{1}{\text{Sin}^2(x)}\times[\text{Sin}^2(x) \mid \text{Cos}^2(x)]$
(5) Use the trig identity under Pythagorean Identities $\text{Sin}^2(x) \mid \text{Cos}^2(x) \quad 1$	$= -\frac{1}{\text{Sin}^2(x)} = -\frac{1}{\text{Sin}(x)}\times\frac{1}{\text{Sin}(x)}$
(6) Use the trig identity $\text{Csc}(x) \quad \frac{1}{\text{Sin}(x)}$	$= \text{Csc}(x)\times\text{Csc}(x) \quad \text{Csc}^2(x)$
Therefore, $\frac{d}{dx}\text{Cot}(x) \quad \text{Csc}^2(x)$	

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